

Fall 2022

**Memorandum**

**Team Members Information**

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**Math Methods**

PETE 5355

**1- Selected Article:**

The journal article was selected from Results in Physics Journal, and it was published in 2016. The article has a set of ordinary differential equations representing initial value problems (ODE-IVP).

The article studies the heat and mass transfer effects on revolving flow of Maxwell fluid due to unidirectional stretching surface and discuss mass transfer influence of binary chemical reaction with activation energy on rotating flow of Maxwell fluid over a stretchable surface. In brief, the main objective of the selected article was to understand the concept of activation energy is usually applicable in areas pertaining to geothermal or reservoir engineering and mechanics of water and oil emulsions.

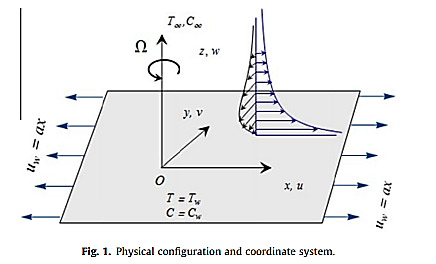
Article details are:

1. Title: Boundary layer flow of Maxwell fluid in rotating frame with binary chemical reaction and activation energy
2. Authors: Z. Shafique, M. Mustafa and A. Mushtaq
3. Journal: Results in Physics
4. Publisher: Elsevier
5. Publication Date: September 2016

**2- Physical phenomena, examined/model assumptions and model derivation:**

a) Physical phenomena:

It describes the consequences of binary chemical reaction with Arrhenius activation energy of binary chemical reactions on rotating flow of Maxwell fluid over a stretchable surface. Activation energy can be realized as energy barrier that separates two minima of potential energy (of the reactants and products of a reaction) which has to be overcome by reactants to initiate a chemical reaction.



*Figure 1: Physical configuration and coordinate system*

b) Model assumptions:

1. Three-dimensional flow of an incompressible Maxwell fluid over an elastic surface located in xy-plane.
2. The fluid resides in the space z 0.
3. The surface is stretched in the x direction with the linearly varying velocity of the form , which induces flow in the neighboring layers of the fluid.
4. The rotating fluid has a constant angular velocity of .
5. The surface temperature is kept constant while and are the ambient values of temperature and solute concentration respectively.
6. Model derivation steps:

1. The conventional boundary layer approximations expressed in partial differential equation form as following:

5

4

The boundary conditions are:

3

2

1

2. The PDE *(Equation 1 to Equation 5)* is converted to a system of first order differential equations (ODE) using shooting approach as following:

7

6

8

9

The transformation conditions are:

Then *(Equation 6 to Equation 9)* are converted by conventional shooting approach into a system of first order differential equations (ODE) is:

, , , , , , , and

So that we obtain the following by:

**3- ODE(s) and boundary condition(s):**

The system of first order differential equations (ODE) is:

, , , , , , , and

So that we obtain the following by:

The initial boundary conditions (IVP) are:

**4- Variables and their physical meaning:**

The variables mentioned in the paper are mentioned in the table below with their respective physical meaning.

|  |  |
| --- | --- |
| **Variable** | **Physical Meaning** |
|  | Solute concentration at surface |
|  | Surface temperature |
| k | Thermal conductivity |
| D | Solute diffusivity |
| ρ | Fluid density |
|  | Specific heat |
| Ω | Angular velocity |
|  | Centrifugal force |
|  | Pressure gradient |
|  | Arrhenius function |
|  | Boltzmann constant |
|  | Reaction rate |
| n | Fitted rate constant |
| S | Extra stress tensor |
|  | Fluid relaxation time |
|  | First Rivlin-Ericksen tensor |
| η | Dimensionless vertical distance |
| λ | Rotation parameter |
| β | Deborah number |
|  | Prandtl number |
|  | Schmidt number |
| E | Non-dimensional activation energy |
| δ | Temperature difference parameter |
| σ | Dimensionless reaction rate |
|  | Nusselt number |
|  | Sherwood number |
|  | Wall heat flux |
|  | Wall mass flux |
|  | Local Reynolds number |

*Table 1: Variables and their physical meaning*

**5- Model equation classification and boundary conditions:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Equation** | | **Dependent and independent variables** | **Linear or nonlinear** | **Homogenous or non-homogenous** | **Order** | **Steady state or non-steady state** | **Deterministic or probabilistic** | **Microscopic, multiple gradients, maximum gradient or macroscopic** |
| **Partial Differential Equations** | | | | | | | | |
|  | | Dependent: u, v and w  Independent: x, y and z | Linear | Non-homogeneous | First | Steady state | Deterministic | Microscopic |
|  | | Dependent: u, v for constant values of Independent: x, y and z | Non-linear | Non-homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | Dependent: u, v for constant values of Independent: x, y and z | Non-linear | Non-homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | Dependent: T for constant values of Independent: x, y and z | Linear | Homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | Dependent: C for constant values of Independent: x, y and z | Linear | Non-homogeneous | First | Steady state | Deterministic | Microscopic |
| **Boundary Conditions** | | | | | | | | |
| **Equations** | **Boundary conditions** | **Dependent and independent variables** | **Linear or nonlinear** | **Homogenous or non-homogenous** | **Order** | **Steady state or non-steady state** | **Deterministic or probabilistic** | **Microscopic, multiple gradient, maximum gradient or macroscopic** |
|  | Dirichlet | N/A | Linear | N/A | N/A | N/A | N/A | N/A |

*Table 2: Partial differential equations and boundary conditions*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Equation** | | **Dependent and independent variables** | **Linear or nonlinear** | **Homogenous or non-homogenous** | **Order** | **Steady state or non-steady state** | **Deterministic or probabilistic** | **Microscopic, multiple gradients, maximum gradient or macroscopic** |
| **Ordinary Differential Equations** | | | | | | | | |
|  | | N/A | Non-linear | Non-homogeneous | Third | Steady state | Deterministic | Microscopic |
|  | | N/A | Non-linear | Non-homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | N/A | Linear | Homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | N/A | Linear | Homogeneous | Second | Steady state | Deterministic | Microscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: for constant  Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: for constant  Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
|  | | Dependent: for constant  Independent: | Linear | Non-homogeneous | First | Non-steady state | Deterministic | Macroscopic |
| **Boundary Conditions** | | | | | | | | |
| **Equations** | **Boundary conditions** | **Dependent and independent variables** | **Linear or nonlinear** | **Homogenous or non-homogenous** | **Order** | **Steady state or non-steady state** | **Deterministic or probabilistic** | **Microscopic, multiple gradients, maximum gradient or macroscopic** |
|  | Cauchy | N/A | Linear | N/A | First | N/A | N/A | N/A |
| **Initial Conditions** | | | | | | | | |
| **Equations** | **Boundary conditions** | **Dependent and independent variables** | **Linear or nonlinear** | **Homogenous or non-homogenous** | **Order** | **Steady state or non-steady state** | **Deterministic or probabilistic** | **Microscopic, multiple gradients, maximum gradient or macroscopic** |
|  | Dirichlet | N/A | Linear | N/A | N/A | N/A | N/A | N/A |

*Table 3: Ordinary differential equations, boundary conditions and initial conditions*

The equations are either macroscopic or microscopic due to the following reasons:

a) When the dependent variables are function of space and position. Therefore, the model is macroscopic model.

b) When is no empirical coefficient in the model equations. Therefore, the model is not a multiple or maximum gradient.

c) Model equations involve a phenomenological approach, and the dependent variables aren’t function of space and position. Hence, it is a microscopic model.

**6- ODE Non-dimensional form:**

The non-dimensional quantities are:

The previous non-dimensional quantities were converted to the ODE as we mentioned before along with equations *(Equation 1 to Equation 5)* to a system of first order differential equations (ODE) using shooting approach as following:

The system of dimensional first order differential equations (ODE) is: